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DEPTH AND POSITION ERROR BUDGETS FOR MULTIBEAM ECHOSOUNDING

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Abstract

Depth error budgets are commonplace for single-beam echosounders, but less so for their multibeam counterparts. Position error budgets for single-beam echosounders are seldom prepared, relying rather, on the positioning system accuracy specifications. Multibeam echosounders (MBES), in addition to having their own measurement errors, have errors resulting from the measurement inaccuracies of the additional sensors, which are needed in order to compute the depth and position of each sounding. This paper presents the general equations that relate measured quantities from all sensors to reduced depths and positions using MBES systems. The error equations are derived from these, using the method of propagation of errors. A simple model for sound speed profile errors is derived, and an empirical method for estimating sounder range and beam angle errors is presented. Total error budgets for depth and position are summarized and presented using small angle approximations.

1. INTRODUCTION

Like many other Hydrographic Offices, the Canadian Hydrographic Service (CHS) has recently acquired and begun surveying with MBES. After initially being overwhelmed with massive amounts of data and being dazzled by impressive three-dimensional displays of bathymetry, CHS realized that there was a need to quantify the accuracy of these modern systems, through total error budgets of depth and position. This need arises from the necessity to compare, and perhaps integrate, multibeam data with data collected using the traditional, single-beam echosounder technology. Preparing error budgets can also give an indication of where improvements can be made in these systems in order to increase the accuracy of depths or positions. Estimation can also be made of the suitability of a particular

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MBES system configuration to a survey task at hand. This paper develops these total error budgets from the general equations for depth and position.

2. BACKGROUND

Hydrographic surveyors around the world have for years prepared depth error budgets for single beam echosounder surveys in order to ensure that IHO standards, or the specifications laid out in their own survey instructions, can be met. The error sources for single-beam echosounders and depth reductions, along with the methodology for producing depth error budgets, have been well documented [4]. There have also been attempts to improve on the IHO standard [2] for depth measurement accuracy, because of the discontinuity at 30 metres depth [3].

MBES systems have several sources of error in common with single-beam echosounders, but also have additional sources of measurement error and errors due to measurements made by other sensors. Because of these additional sources of error, a total depth error budget of MBES systems is required.

In the past, it was assumed, perhaps lightly, that the accuracy of position of a single-beam echosounder depth on the seafloor was equivalent to the precision of the positioning system. This assumption was made for several reasons:

- calibration procedures were followed to ensure the accuracy of the positioning system,
- the beamwidth of the single-beam echosounder was typically wide enough to absorb the positioning errors caused by roll and pitch,
- the positioning system antenna was located close enough (within the positioning system error) to the transducer to be considered coincident,
- because of the cost of a gyrocompass, launches seldom logged heading information, and
- the precision of the positioning system and the sounder resolution were never good enough to allow accurate estimation of positioning system latency.

However, there has been a revolution in both position and depth measurement capabilities, which is forcing Hydrographic Offices to examine position error budgets as well. Because a depth obtained by a MBES can be at some distance from the positioning system, and the positioning of that sounding is dependent on the vessel's attitude sensors, a total position error budget for MBES systems is also required.

3. DEPTH AND POSITION EQUATIONS FOR MBES

In order to derive the error budgets for MBES systems, the relationships between the measured values and the derived quantities of depth and position must first be established. By applying the method of propagation of errors [6] to these relationships, approximate depth and position error equations can be developed.

Figure 1 shows the various sensors used in MBES systems in a boat-fixed coordinate system (the body frame). The body frame chosen here is the right-handed coordinate system used by the TSS 335B heave, roll and pitch (HRP) sensor, also called a vertical reference unit or VRU.

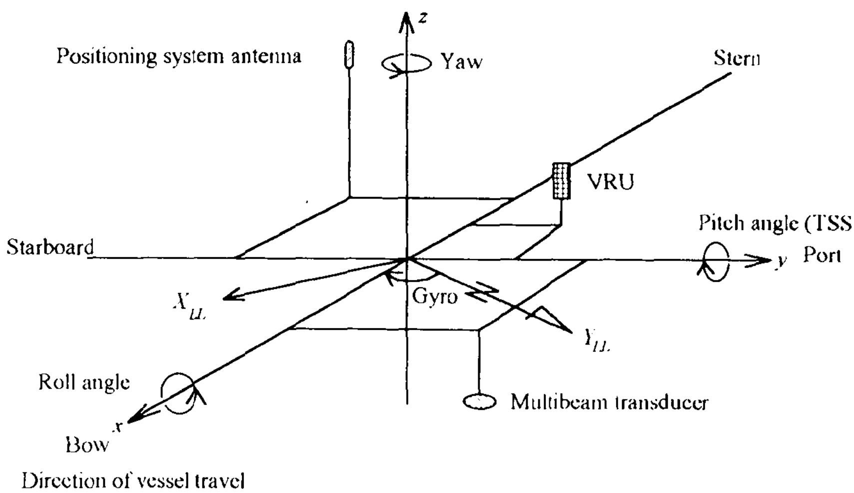

FIG. 1.- Vessel-based, right-handed coordinate system *(local-level also shown).*

The arrows at the end of each axis indicate the positive direction. A North arrow is shown to indicate that the vessel is heading roughly north-east. The arrows around each axis indicate the direction of positive rotation. Although the pitch angle in a right-handed coordinate system should be positive when the bow of the vessel is down, the TSS 335B negates this value so that the output pitch angle is positive for bow-up vessel attitudes (a maritime convention).

For each sounder ping, several depth soundings are obtained beneath and to either side of the vessel. A latitude, longitude and depth is needed for each beam across this swath (lateral coverage from one sounder ping). The sounding position coordinates can be calculated by adding the offset coordinates of each sounding from the positioning system antenna to the latitude and longitude of the antenna as determined by the positioning system receiver. The method will be described later. The depth of each sounding is calculated from the slant range measurement (travel-time) and the receipt angle, which will be discussed next.

Sounder system equations

The cross-track distance and depth can be calculated from the range (determined by measuring two-way travel time) and beam angle as shown in Figure 2.

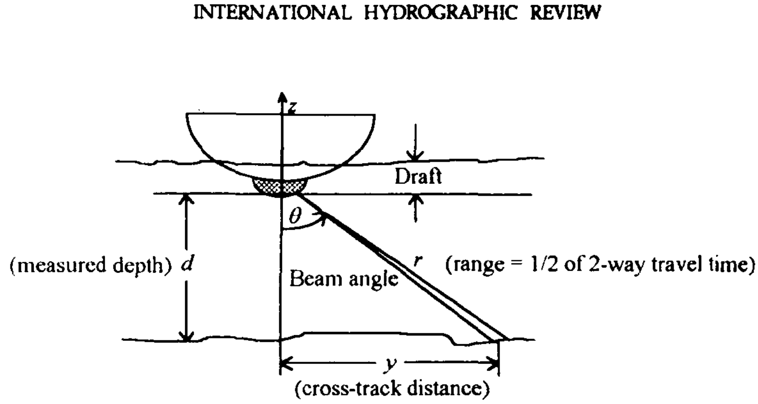

FIG. 2.- Position and depth calculation for a MBES system.

Using simple geometry, the cross-track distance, y and the depth below the transducer, d can be calculated by:

$$y = r \sin \theta \tag{1}$$

$$d = r \cos \theta = -z \tag{2}$$

The range, r and beam angle, θ are the geometric distance and direction from the transducer to the point on the seafloor where the centre of the beam makes contact. The along-track coordinate (x) is zero if the transducer is not pitched. Referring to Figure 1, it should be apparent that these coordinates can be corrected for roll, pitch and heading angles by rotating the vector in the body frame about the three orthogonal coordinate axes. This can be written using a matrix equation of the form:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{L} = R\ (\alpha,\ P,\ R) \begin{bmatrix} 0 \\ r \sin \theta \\ -\ r \cos \theta \end{bmatrix}_{B} \tag{3}$$

where R is the roll angle of the body frame, P is the pitch angle of the body frame, α is the gyrocompass heading of the body frame (from North), r is the geometric range from the transducer to the bottom and α is the geometric beam angle from the nadir with positive beams to port.

The subscripts LL and B on the vectors represent the local-level and body reference frames respectively. The local-level coordinate system is also a right-handed coordinate system: z-axis up, y-axis North and x-axis East (see Figure 1). The local-level vector enables the calculation of three-dimensional coordinates in the global reference frame using the coordinates of the antenna and the coordinate offsets of the antenna from the transducer which will be discussed later. The negative sign for the z coordinate in the body frame ($r \cos \alpha$) is because the z-axis is defined as pointing up. Treating the third coordinate as depth, this value becomes positive as shown in Equation 2.

The large capital "**R**" in Equation 3 is a rotation matrix operator. There are in fact separate rotations about each of the three orthogonal axes (see Equation 4): about the x-axis (axis 1 - denoted by 1 subscript in $\mathbf{R}_1$) by the negative of the roll angle, about the y-axis (axis 2 - denoted by 2 subscript in $\mathbf{R}_2$) by the pitch angle and about the z-axis (axis 3 - denoted by 3 subscript in $\mathbf{R}_3$) by the negative of 90° minus the heading angle. The rotation about the x-axis uses a negative angle in order to rotate the rolled vector into a level system. The pitch angle rotation is positive because the maritime convention for pitch angles is negative in a right-handed coordinate system. The rotation about the z-axis is opposite to the measurement direction of the heading angle and is measured from the y-axis (North). For this reason the heading angle must be subtracted from 90° and then negated, i.e. α-90°. The total rotation matrix sequence is given as follows:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_L = R_3\,(\alpha - 90)\; R_2\,(P)\; R_1\,(-R) \begin{bmatrix} 0 \\ r\sin\theta \\ -\,r\cos\theta \end{bmatrix}_B \tag{4}$$

Care must be taken to ensure that the roll and pitch angles used in Equation 4 are the Euler angles (angles measured in a rotated coordinate frame as defined by the Tate-Bryant Convention) and have not been transformed into the local-level coordinate frame by the attitude sensor. The TSS 335B for example uses the following convention:

$$\sin(R) = \sin(\psi)\,\cos(P) \tag{5}$$

where ψ is the Euler roll angle. The Euler pitch angle, P is the negative of maritime pitch convention as discussed above (which has no effect on Equation 5). Because the gyrocompass is gimballed and oriented North, heading is already in the local-level coordinate system and therefore needs no correction. Hereafter, the roll angle is assumed to be the Euler roll angle given by:

$$\psi = \arcsin\left(\frac{\sin(R)}{\cos(P)}\right) \tag{6}$$

In other words, the pitch angle is assumed to be small enough that the Euler roll angle can be approximated by the output roll angle without significant error. The form of the rotation matrices can then be given as:

$$R_3(\alpha-90)\,R_2(P)R_1(-R) = \begin{bmatrix} \sin\alpha & -\cos\alpha & 0 \\ \cos\alpha & \sin\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos P & 0 & -\sin P \\ 0 & 1 & 0 \\ \sin P & 0 & \cos P \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos R & -\sin R \\ 0 & \sin R & \cos R \end{bmatrix} \tag{7}$$

When these are multiplied together, the combined orthogonal rotation matrix is:

$$R(\alpha,P,R)=\begin{bmatrix} \sin\alpha\cos P & -\cos\alpha\cos R-\sin\alpha\sin P\sin R & \cos\alpha\sin R-\sin\alpha\sin P\cos R \\ \cos\alpha\cos P & \sin\alpha\cos R-\cos\alpha\sin P\sin R & -\sin\alpha\sin R-\cos\alpha\sin P\cos R \\ \sin P & \cos P\sin R & \cos P\cos R \end{bmatrix} \qquad (8)$$

Premultiplying the vector on the right-hand side of Equation 3 by this matrix yields the following three equations:

$$x_{LL}=-r\sin\theta(\cos\alpha\cos R+\sin\alpha\sin P\sin R)-r\cos\theta(\cos\alpha\sin R-\sin\alpha\sin P\cos R) \qquad (9)$$

$$y_{LL}=r\sin\theta(\sin\alpha\cos R-\cos\alpha\sin P\sin R)+r\cos\theta(\sin\alpha\sin R+\cos\alpha\sin P\cos R) \qquad (10)$$

$$z_{LL} = r\sin\theta\cos P\sin R - r\cos\theta\cos P\cos R \qquad (11)$$

Depths from MBES systems

The depth and position equations will be examined separately, beginning with the equation for measured depth. Rearranging Equation 11 yields the following:

$$z_{LL} = r\cos P\,(\cos\theta\cos R - \sin\theta\sin R) \qquad (12)$$

Recalling a trigonometric identity for the sums of angles:

$$\cos(A+B)=\cos A\cos B-\sin A\sin B \qquad (13)$$

Equation 11 can be simplified to the following:

$$z_{LL}=-r\cos P\cos(\theta+R) \qquad (14)$$

Substituting Equation 14 into Equation 2 results in Equation 15, which is the principal equation for MBES depth calculation from the measured quantities of range, beam angle, roll and pitch.

$$d = r\cos P\cos(\theta + R) \qquad (15)$$

Note that d is in a local-level coordinate system by definition (measured perpendicular to a level surface - i.e. the water surface). This value is the depth below the transducer at the instant of measurement, but only when r is the true, straight-line <u>geometric</u> distance to the seafloor and θ is the <u>geometric</u> beam angle as shown in Figure 2. In order to get these geometric quantities from what was measured by the transducer, corrections for ray-bending and propagation effects due to the sound speed profile in the water column must be made. The measured roll and pitch angles from the VRU will also need correction since the VRU alignment to the body frame may be different from that of the transducer. Although there are both roll and beam angle terms in the brackets it should be stressed that some MBES systems steer the receive beams in real-time to compensate for smearing due to excessive roll and to ensure that a uniform coverage is obtained. The roll term has been left in Equation 15 as a reminder that there is an error contribution to beam angle from any roll measurement error. The errors in these quantities will be discussed in Section 4.

So far, only errors which affect the sounder system (echo-sounder and angular motion sensor) or those errors which affect the measurement of the vertical distance from the transducer to the seafloor (sound speed errors) for each depth across a swath, have been discussed. There are several other potential sources of error, independent of the sounder, which affect accuracy of the final reduced depth. Two of these errors - dynamic draught and heave - are dependent on the sounding platform and, in the case of heave, on the location of the attitude sensor in relation to the transducer.

Dynamic draught

Dynamic draught is the instantaneous depth of the transducer below the mean water level and is made up of three components as shown in the following equation:

$$dyn\_draught = draught - squat - load \tag{16}$$

where *draught* is the depth of the transducer below the water level when the vessel is at rest, *squat* is the change in the draught with changes in vessel speed and *load* is the change in draught over time, e.g. because of fuel consumption.

Draught is the vertical distance between the water level and the measurement centre of the transducer when the vessel is at rest. The draught must therefore be determined for each survey platform, perhaps on a daily basis. In some cases, determination may be a direct measurement from the water level to the transducer and in another case, it may require reference to draught marks which have a known relationship to the transducer.

Squat is defined in the Mariner's Handbook as the "difference between the vertical positions of a vessel moving and stopped. It is made up of settlement and change of trim." Settlement is the general lowering of a vessel due to the change in the level of the water around her and is a function of the depth of water and the speed of the vessel. The effect of increasing speed on planing-hulled vessels (common for small boats) is to cause them to lift out of the water. For displacement-hulled vessels, forward motion generally causes the stern to settle.

Changes in load will cause the water line measurement to be different on a daily basis and may change non-linearly over the course of a day. The load change, as a function of time, could be approximated by a linear function if the draught at rest was measured at the start and end of each day. It may also be possible to monitor fuel and water levels and model the load change as a function of these parameters.

$$\Delta x_s = -r\sin\theta(\cos\alpha\cos R + \sin\alpha\sin P\sin R) - r\cos\theta(\cos\alpha\sin R - \sin\alpha\sin P\cos R) \tag{9}$$

$$\Delta y_s = r\sin\theta(\sin\alpha\cos R - \cos\alpha\sin P\sin R) + r\cos\theta(\sin\alpha\sin R + \cos\alpha\sin P\cos R) \tag{10}$$

Rearranging these two equations, gives the following form:

$$\Delta x_s = -r\cos\alpha(\sin\theta\cos R + \cos\theta\sin R) + r\sin\alpha\sin P(\cos\theta\cos R - \sin\theta\sin R) \tag{23}$$

$$\Delta y_s = r\sin\alpha(\sin\theta\cos R + \cos\theta\sin R) + r\cos\alpha\sin P(\cos\theta\cos R - \sin\theta\sin R) \tag{24}$$

Using the trigonometric identity given by Equation 13 for the sums of angles, and another trigonometric identity given by the following:

$$\sin(A + B) = \sin A\cos B + \cos A\sin B \tag{25}$$

after substitution and simplification Equations 23 and 24 become, respectively:

$$\Delta x_s = -r\cos\alpha\sin(\theta + R) + r\sin\alpha\sin P\cos(\theta + R) \tag{26}$$

$$\Delta y_s = r\sin\alpha\sin(\theta + R) + r\cos\alpha\sin P\cos(\theta + R) \tag{27}$$

As was the case for depth, the roll and pitch angles are corrected for the misalignment of the transducer, but for these equations the gyrocompass heading must also be corrected for transducer yaw misalignment, since this will affect the location of the sounding on the seafloor. This component of position is independent of the location of the attitude sensor, positioning system accuracy and data logging.

Relative transducer position

The offsets of the positioning system antenna from the transducer, also in the local-level coordinate system, are added to the sounding coordinate offsets. These offsets can be calculated by pre-multiplying a vector of transducer-antenna coordinate offsets in the body frame by the rotation matrix given in Equation 8. The local-level coordinates of these offsets are given as:

$$\Delta x_a = x\sin\alpha\cos P - y(\cos\alpha\cos R + \sin\alpha\sin P\sin R) + z(\cos\alpha\sin R - \sin\alpha\sin P\cos R) \tag{28}$$

$$\Delta y_a = x\cos\alpha\cos P + y(\sin\alpha\cos R - \cos\alpha\sin P\sin R) - z(\sin\alpha\sin R + \cos\alpha\sin P\cos R) \tag{29}$$

These coordinate offsets represent a vessel-specific source of error, since the separation coordinates are a function of where the antenna and transducer are located. The roll, pitch and heading angles are those measured by the sensors and are now independent of transducer misalignment. Because these offsets are independent of the sounder system, they only need to be calculated once for each sounder ping.

The common practice is to measure the coordinate offsets of all sensors from some common reference point - the coordinate centre of the body frame. This point may be a convenient point on the ship such as the centre of roll or the centre of pitch, or it may be chosen to be at one of the sensors. In any case, the coordinate offsets represent three-dimensional vectors in the body frame. As such, vectors from the centre of the body frame to any two sensors may be subtracted in order to determine the vector between the two sensors. The errors in this vector sum are then simply the quadratic summation of the errors in the determination of each sensor's coordinates.

Relative position-time displacement

The final coordinate offset to be dealt with is due to timing offsets between the position system and the transducer depth measurement. The coordinate offset will always result in an x-axis displacement in the body frame and is calculated as the product of the time offset (or latency), Δt and the ship's speed over ground, SOG. This vector can be rotated into the local level coordinate system using Equation 3 as:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_L = \mathbf{R}\ (\alpha, P, R) \begin{bmatrix} \Delta t SOG \\ 0 \\ 0 \end{bmatrix}_B \tag{30}$$

Multiplying the vector on the right-hand side of Equation 30 by the rotation matrix given in Equation 8, we get the following three coordinate offsets due to positioning system latency:

$$\Delta\ x_t = \Delta t SOG \sin\alpha \cos P \tag{31}$$

$$\Delta\ y_t = \Delta t SOG \cos\alpha \cos P \tag{32}$$

$$\Delta\ z_t = \Delta t SOG\ \sin P \tag{33}$$

These offsets are logging-system specific, but also depend on ship dynamics. The equations also show why it is difficult, using a patch test, to separate pitch misalignment from positioning system latency. The z-coordinate offset due to latency, which will be negligible for small pitch angles, is ignored in this analysis.

4. DEPTH ERROR EQUATIONS

In the following sections, the various sources of error in the right-hand side of Equation 15 will be examined. Then, a method to map these error contributions into their depth measurement error components will be given. Measured range and beam angles contain measurement noise and are also affected by errors in the sound speed, both at the transducer face, in the case of beams steered non-orthogonally to the transducer, and throughout the entire water column. The measured roll and pitch angles, as determined from the VRU, also contain measurement noise and may be affected by a time delay from the VRU. In addition, there will be errors in the determination of the roll and pitch offset angle of the sonar head orientation from that of the VRU. These errors can be determined from a calibration procedure such as a patch test [1]. The errors which affect the echosoundings themselves (i.e. the range and beam angle) are examined first.

Geometric range

The measured range (half the total travel time multiplied by the average measured sound speed) can be corrected for the true (geometric mean) sound speed, v, by a simple ratio of sound speeds as follows:

$$r = r_m \left(\frac{v}{v_m} \right) \tag{34}$$

From Equation 34, errors in the determination of geometric range can be seen to come from both range measurement errors and from errors due to an imperfectly known sound speed profile. Applying the method of propagation of errors, the total variance of true geometric range is:

$$\sigma_r^2 = \left(\frac{\partial_r}{\partial_{r_m}} \right)^2 \sigma_{r_m}^2 + \left(\frac{\partial_r}{\partial_{v_m}} \right)^2 \sigma_{v_m}^2 \tag{35}$$

where the variance of range and sound speed are assumed to be the square of the measurement errors in these quantities. It is assumed that all measurements are uncorrelated random variables, with normally distributed errors, such that the rules of statistics apply. The true sound speed is without error (i.e. zero variance). The first term in Equation 35, after substituting the partial derivative, gives:

$$\sigma_{r_1}^2 = \left(\frac{v}{v_m} \right)^2 \sigma_{r_m^2} \approx \sigma_{r_m^2} \tag{36}$$

The second term in Equation 35, after substituting the partial derivative, gives:

$$\sigma_{r_2}^2 = \left(\frac{-v.r_m}{v_m^2} \right)^2 \sigma_{v_m^2} \approx \left(\frac{r_m}{v_m} \right)^2 \sigma_{v_m^2} \tag{37}$$

Both equations are only approximations since the measured sound speed will be different from the true sound speed, due to measurement limitations and spatial or temporal changes in the sound speed. If the difference is small however, the ratio is very nearly 1, because the speed of sound in water is between 1400 and 1500 m/s. The total variance in range will be the sum of Equations 36 and 37 or:

$$\sigma_r^2 \approx \sigma_{r_m}^2 + \left(\frac{r_m}{v_m}\right)^2 \sigma_{v_m}^2 \qquad (38)$$

The total error in range can then be obtained by taking the square-root of this variance.

Geometric beam angle

Assessing the errors in the beam angle is much more complicated than for range. This is due to the fact that the beam bends with the sound speed gradient as defined by Snell's Law. At each sound speed interface (see Figure 3), the sine of the incident and refracted beam angles are related to the ratio of the sound speeds on either side of the interface as follows:

$$\sin \theta' = \sin \theta \, \frac{v'}{v} \qquad (39)$$

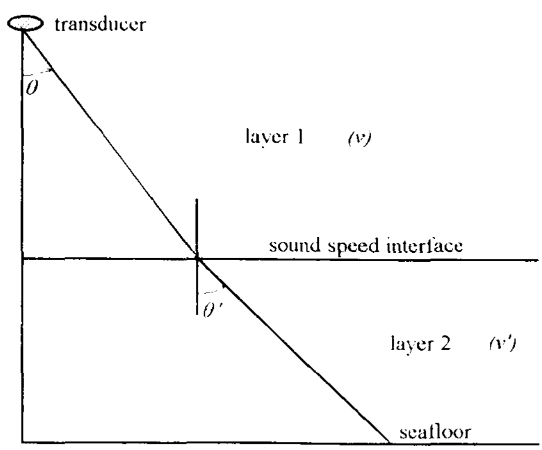

FIG. 3.- Beam angle bending at a sound speed interface as defined by Snell's Law.

Since the sound speed profile is sampled discretely there will be many such interfaces over the entire water column. This situation leads to a need for some form of approximation. As a simplification, the geometric beam angle can be given as the sum of the measured beam angle and a small angular correction for the true sound speed profile as follows:

$$\theta = \theta_m + \theta_v \tag{40}$$

The total error in beam angle is then made up of two components: one due to beam measurement error and the other due to errors in the sound speed profile, represented by the following equation:

$$\sigma_\theta^2 \approx \sigma_{\theta_m}^2 + \sigma_{\theta_v}^2 \tag{41}$$

Sound speed errors

There will be errors introduced into the final coordinates (depth and position) as a result of imperfectly known sound-speed profiles. Such errors may come from errors in measurement of the sound speed, due to velocimeter measurement limitations, errors in the determination of the depth at which the sound speed measurements were taken or errors due to spatial or temporal changes in the sound-speed profile. Equation 38 shows how sound-speed profile errors propagate into an error in range. Errors in beam angle result from incorrect steering due to errors in the sound speed at the transducer (for beams which have to be steered non-orthogonally to the transducer face) and from errors in the geometric correction to the beam angle due to sound-speed profile errors. These errors are examined next.

Errors in beam angle due to sound speed profile errors

By differentiating Equation 39, the error in θ' due to imperfect knowledge of sound speed v' at a sound speed layer boundary is:

$$\cos\theta' \, d\theta' = \sin\theta \, \frac{dv'}{v} \tag{42}$$

where dv' is the error in the sound speed profile and $d\theta'$ is the resultant error in the beam angle at the sound speed interface. Assuming that the thickness of both layers is the same (see Figure 4), then a reasonable approximation for the error in the geometric beam angle due to errors in the mean sound speed profile is half that of the error at the middle boundary. Using this assumption and moving terms to the right-hand side of Equation 42 gives:

$$d\theta_{vp} \approx \tan\theta\left(\frac{dv}{2v}\right) \tag{43}$$

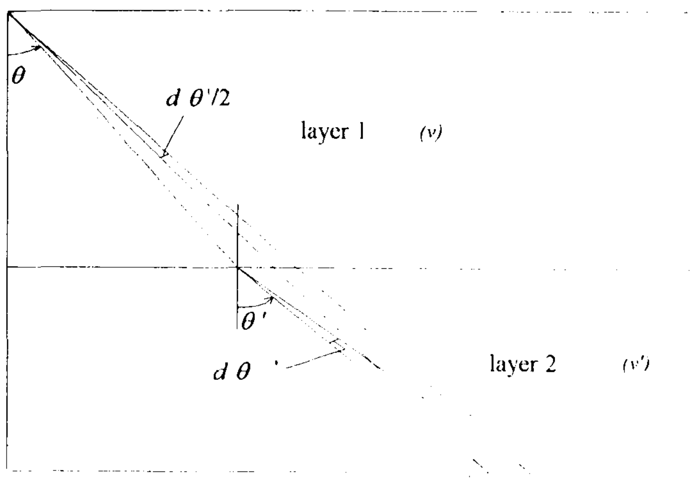

FIG. 4.- Sound speed error model using two equal layers.

The variance in the geometric beam angle due to sound speed profile errors may be represented by the following approximation:

$$\sigma^2_{\theta_{vp}} \approx \sigma^2_v \left(\frac{\tan \theta}{2v} \right)^2 \tag{44}$$

Beam steering - surface sound speed errors

In order to steer the receive beam, the sound speed at the transducer face must be precisely known. Errors in the knowledge of this sound speed will result in incorrect steering due to an incorrect calculation of the timing delay for each transducer element.

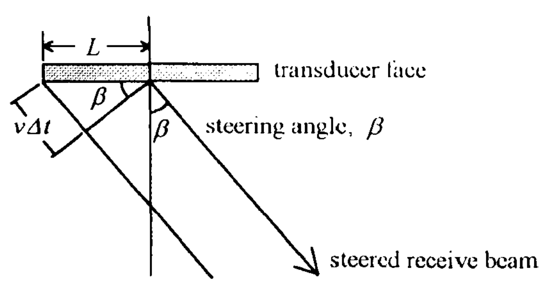

FIG. 5.- Beam steering from a flat transducer.

In order to steer a beam non-orthogonally to the transducer face, the reception of the transducer elements must be sequenced. The elements further from the wave front are triggered at a later time and elements closer to the wave front are triggered sooner, such that the wave front remains coherent. Some MBES use curved transducers, because they are less dependent on precise sound speed at the transducer. Beam steering is, however, easier to visualize using a flat transducer. From Figure 5, it can be seen that for a flat transducer to steer a beam non-orthogonally to the transducer face by an angle β, the length of the transducer segment used for beam forming, L, and the speed of sound in water, v, are needed to calculate the required time delay, Δt. The following relationship holds:

$$\sin\beta = \frac{v\Delta t}{L} = v\frac{\Delta t}{L} \tag{45}$$

By differentiating Equation 45 and performing a substitution, the error in the angle β caused by an error in the surface sound speed is given by:

$$d\beta = \frac{1}{\cos\beta}\frac{\Delta t}{L}dv = \frac{\sin\beta}{v\cos\beta}dv = \frac{\tan\beta}{v}dv \tag{46}$$

where dv represents the error in sound speed at the transducer face. The steering angle, β is the difference between the steered beam angle, θ and the angle normal to the transducer face in the centre of the symmetric transducer element array used for beam forming, δ.

The error in beam steering due to imprecise surface sound speed, for non-orthogonally-steered beams, can then be given by the following relationship:

$$d\theta_{v_s} = \frac{\tan(\theta-\delta)}{v_s}dv_s \tag{47}$$

The variance can then be approximated by:

$$\sigma\theta_{v_s 2} \approx \left(\frac{\tan(\theta-\delta)}{v_s}\right)^2 \sigma v_s^2 \tag{48}$$

Errors in sound speed for orthogonally steered beams will cause a change in the beamwidth but not its direction, since they can be steered using a symmetric array of transducer elements.

The effects of errors in sound speed on the beam angle can be accounted for by adding two beam angle variances, which account for beam steering and sound speed profile errors as:

$$\sigma^2_{\theta_v} = \sigma^2_{\theta_{v_p}} + \sigma^2_{\theta_{v_s}} \tag{49}$$

There are other contributions to the total variance in beam angle, which will be given later. The errors in the measurement of range and beam angle (the measurement components of Equations 38 and 41) are discussed next.

Sounder measurement errors

Measurement of range and beam angle for multibeam echosounders is more complex than for vertical-incidence echosounders. As a result, errors in these measurements are due to several factors including: incident angle with the seafloor, range sampling resolution and transmit and receive beamwidths. At least one MBES manufacturer has produced a model of the measurement errors in range and beam angle, dependent on mode of detection, some sounder system parameters, and seafloor slope [5]. Development of error models for other multibeam echosounders is still needed. An empirical method for estimating range and beam angle measurement errors is given later in this section.

Transducer misalignment angles

The transducer may not be oriented exactly the same as the VRU, so measured roll and pitch angles should be adjusted to reflect the "real" roll and pitch angles experienced by the transducer in the local-level coordinate system using:

$$R = R_m + \Delta R \tag{50}$$

$$P = P_m + \Delta P + P_s \tag{51}$$

where the first correction terms represent the misalignment angles determined from a patch test. The extra term in Equation 51 accounts for the pitch angle of the mechanical stabilization unit, if one is used. Although the heading misalignment has no effect on depth measurement (no heading term in Equation 15), the value is usually determined in the same way as the roll and pitch misalignment angles (patch test) and can be used to correct the observed transducer heading (see Equation 52) in order to compute the position of the sounding on the seafloor.

$$\alpha = \alpha_m + \Delta\alpha \tag{52}$$

Angular orientation (attitude) errors

Equations 50 and 51 show that errors in roll and pitch can come from both measurement errors of the VRU and errors in the estimation of the transducer alignment angles from the patch test. In the case of mechanically pitch-stabilized systems, an additional pitch angle error will be introduced by the stabilization unit. Errors in the determination of roll will quadratically add to the errors in beam angle, because roll and beam angles are always additive. The total contribution to variance of beam angles and pitch angles can then be given respectively as:

$$\sigma_\theta^2 = \sigma_{\theta_m}^2 + \sigma_{\theta_{v_p}}^2 + \sigma_{\theta_{v_s}}^2 + \sigma_{R_m}^2 + \sigma_{\Delta R}^2 \tag{53}$$

$$\sigma_P^2 = \sigma_{P_m}^2 + \sigma_{\Delta P}^2 + \sigma_{P_s}^2 \tag{54}$$

where the contributions to beam angle variance come from beam angle measurement errors, errors in beam angle due to sound speed profile errors and in beam steering due to surface sound speed errors, errors in roll angle measurement and finally errors in the roll misalignment angle of the transducer. The contributions to pitch angle variance similarly come from pitch angle measurement errors, the pitch misalignment angle of the transducer and errors in the pitch stabilization angle (mechanically pitch-stabilized systems only). Second order effects (e.g. the effect of transducer pitch misalignment on roll measurement errors) are assumed to be negligible and have been ignored - a reasonable assumption if the misalignment angles are small.

It should be noted, that errors in the determination of attitude may come from more than just the resolution or measurement precision of the attitude sensor. If the attitude is undersampled (should have at least 10 times the Nyquist frequency for effective time series manipulation), or if the frequency of the attitude is outside the passband of the sensor (as can happen with heave if a vessel is running with a following sea), then errors far greater than the measurement errors of the instrument can occur. In this paper, the attitude is assumed to be well-behaved and within the limits of the sensor, but the reader is cautioned to use manufacturers' specifications for roll and pitch measurement accuracy with care.

For heading, the errors as apparent from Equation 52 will come from transducer misalignment with respect to the gyrocompass, or other heading sensor, and from the gyrocompass measurement errors as given by:

$$\sigma_\alpha^2 = \sigma_{\alpha_m}^2 + \sigma_{\Delta\alpha}^2 \tag{55}$$

Real-time beam steering for roll angle

Some MBES use the roll angle output from the VRU to steer the receive beam angle in real-time. Because of this, the beam angle of each sounding is referenced to the nadir and the outer edges of the swath are uniform even when the ship is rolling. Because there are errors in the roll measurement, however, the beam angle will have these additional errors incorporated as shown in Equation 53. The equations given in the following sections, show only one beam angle term. It should be kept in mind that the roll angle is always part of the actual beam angle and that roll angle error is always part of the total beam angle error.

Mapping sounder system errors into measured depth errors

All of the components of Equation 15, and where their respective error contributions come from, have been discussed. How these errors, or corresponding variances, map into an error in the measured depth is discussed below. Think of Equation 15 as a mapping of the measured parameters into a measured depth. Applying propagation of errors to Equation 15 gives the following equation, which maps the measurement errors into a depth error:

$$\sigma_d^2 = \left(\frac{\partial d}{\partial r}\right)^2 \sigma_r^2 + \left(\frac{\partial d}{\partial \theta}\right)^2 \sigma_\theta^2 + \left(\frac{\partial d}{\partial P}\right)^2 \sigma_P^2 + 2\left(\left(\frac{\partial d}{\partial r}\right)\left(\frac{\partial d}{\partial \theta}\right)\sigma_{r\theta} + \ldots\right) \tag{56}$$

The assumption has been made that all the error sources are normally distributed and act independently. This allows the trailing covariance terms to be dropped. The entire equation can be evaluated at once, or each error source can be considered separately, so as to view in detail each contribution to the total depth error budget.

The first term of Equation 56, upon substituting partial derivatives of Equation 15 with respect to range, reduces to:

$$\sigma_{d_1}^2 = (\cos P \cos \theta)^2 \sigma_r^2 \tag{57}$$

Therefore, to calculate the effect of range error on depth, the beam angle and the amount of pitch on the transducer must also be known. The total range variance comes from the sum of the variance due to range measurement and that due to sound speed errors. The depth contributions can also be examined as two separate components (a sounder measurement contribution and a refraction contribution) which must be quadratically added to obtain the total depth variance contribution due to range.

Similarly, the depth variance due to beam angle (roll angle) errors is given by:

$$\sigma_{d_2}^2 = (r \sin \theta \cos P)^2 \sigma_\theta^2 \tag{58}$$

The total beam angle variance, which includes measurement errors, sound speed effects on beam angle, roll errors and transducer roll alignment errors, is given by Equation 54. Any one component of angular error can be examined separately, to see its effect on the depth.

Finally, the mapping for pitch errors into depth variance is given as follows:

$$\sigma_{d_3}^2 = (r \cos \theta \sin P)^2 \sigma_P^2 \tag{59}$$

The total variance of pitch is given by Equation 55. Note that the units of the standard deviation (square-root of the variance) of depth are metres for each of Equations 57-59. All angular error must therefore be in radians.

Limitations due to beam opening angle

Although not an error as such, the beam opening angle can be a limiting factor in resolving targets of a certain size on the seafloor. Figure 6 illustrates this problem.

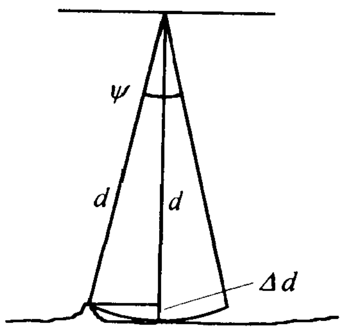

FIG. 6.- Resolution of targets due to beam opening angle.

The return path to the small target at the left-hand side extremity of the beam cone is the same as the vertical path, d. Unless the target is directly under the transducer, a possible error exists as given by the following relation:

$$\begin{aligned} \Delta d &= d - d \cos\left(\frac{\psi}{2}\right) \\ &= d\left[1 - \cos\left(\frac{\psi}{2}\right)\right] \end{aligned} \tag{60}$$

where ψ, is the beam opening angle. The variance is approximately given by the square of this equation or:

$$\sigma_{d_4}^2 \approx \left\{ d\left[1 - \cos\left(\frac{\psi}{2}\right)\right]\right\}^2 \tag{61}$$

Total depth measurement error

The total measured depth error due to the sounder system is given by the root-sum-square (RSS) of the above components as:

$$\sigma_d = \sqrt{\sigma_{d_1}^2 + \sigma_{d_2}^2 + \sigma_{d_3}^2 + \sigma_{d_4}^2} \tag{62}$$

The contributions to the total depth measurement error budget are illustrated in Figure 7.

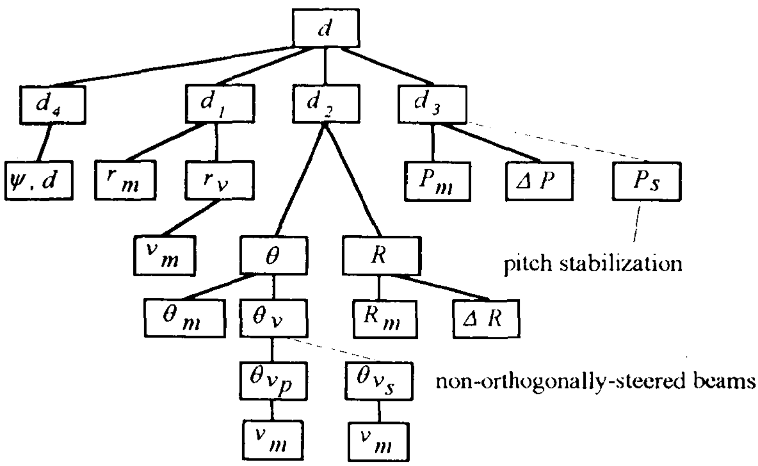

FIG. 7.- Flow diagram showing contributions to measured depth error.

Heave error

The variance of measured heave comes from the manufacturer's specifications for heave accuracy, typically as fixed and variable (function of peak-to-peak heave height) components. Measured heave variance can be calculated by:

$$\sigma_{H_m}^2 = \max\left(a^2, (b * heave)^2 \right) \tag{63}$$

where a is a fixed component in metres and b is a variable component (% of peak-to-peak heave). The variance of roll and pitch induced heave can be calculated by applying the method of propagation of errors to Equation 18, which results in the following equation:

$$\begin{aligned} \sigma_{H_i}^2 = & (x\cos P - y\sin R\sin P - z\cos R\sin P)^2\sigma_P^2 \\ & + (y\cos R\cos P - z\sin R\cos P)^2\sigma_R^2 \\ & + (\sin P)^2\sigma_x^2 + (\sin R\cos P)^2\sigma_y^2 + (1 - \cos R\cos P)^2\sigma_z^2 \end{aligned} \tag{64}$$

Errors due to the measurement of roll and pitch as well as errors in measuring the coordinate offsets between the transducer and the VRU will all contribute to induced-heave error. In order to get the total variance of depth due to heave errors, the heave measurement errors and induced heave errors are quadratically added, which results in:

$$\sigma_H^2 = \sigma_{H_m}^2 + \sigma_{H_i}^2 \tag{65}$$

Note that the variances are expressed entirely in terms of the measured quantities of heave, roll, pitch and the *x*-*y*-*z* offsets between the two sensors. As such, heave error is both a sounding system error (because of the contribution from VRU errors) and a vessel-specific error, dependent on the relative coordinates of the sensors. Thus the transition is made from depth measurement to depth reduction.

Dynamic draught errors

Dynamic draught variance is given by the quadratic sum of error sources as:

$$\sigma_{dyn\_draught} = \sigma_{draught}^2 + \sigma_{squat}^2 + \sigma_{load}^2 \tag{66}$$

Draught, squat and load are values which must be determined for each survey platform and will have error sources peculiar to the characteristics of each vessel. The errors are not the values of draught, squat and loading changes themselves, but the residual errors that remain after correcting the measured depth.

Water level error

Water level errors come from several sources, which will not be described in detail here. The main sources of error are due to water level measurement at the gauge and spatially/temporally predicting the water level at the location of the sounding vessel. There may also be errors due to the method chosen to filter sea-surface waves at the gauge, and due to gauge or sounding vessel timing errors.

Charted depth error

The total depth error may be determined by applying propagation of errors to Equation 20. The charted depth error is given by the RSS of all the above error components as:

$$\sigma_D = \sqrt{\sigma_d^2 + \sigma_H^2 + \sigma_{dyn\_draught}^2 + \sigma_{WL}^2} \tag{67}$$

The first term on the right-hand side of Equation 67 comprises all the error components of depth measurement. The heave term is due to the local sea state, and is also considered to be a measurement error component. The next term makes up the total dynamic draught error, which depends on the vessel, and the final term is for water level error. Figure 8 illustrates the contributions to reduced depth error from all the error sources discussed in this section.

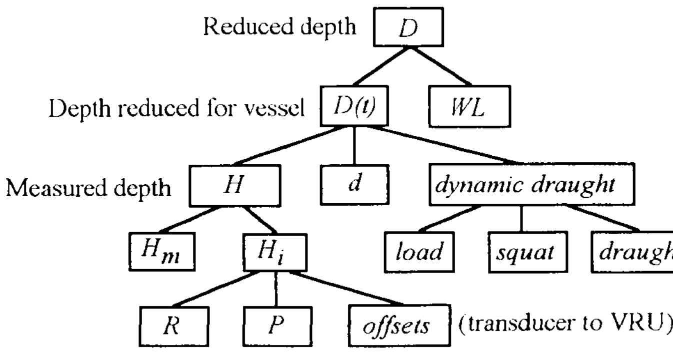

FIG. 8.- Flow diagram of contributions to reduced depth error.

A method for estimating range and beam angle measurement errors

As stated above, depth measurement errors depend on range and beam angle measurement, which both vary with the incident angle each beam makes with the seafloor. In the absence of a MBES measurement error model, it is possible to estimate range and beam angle errors, over a flat seafloor, from depth measurement differences of two coincident swaths. The two lines must be run exactly superimposed, at the same speed and in the same direction, within a short time interval. In this way, it is hoped that water level, refraction and dynamic draught biases will be minimized. Pitch errors are negligible in most cases, and small position errors over a flat seafloor can be tolerated. Thus, the depth difference errors should be due to two independent errors in the measurement of range, beam angle, roll and heave. By moving all but the range and beam angle error components to one side of the equation, the following equation can be formed:

$$\sigma_{\Delta d}^2 - 2\sigma_H^2 - 2(r\sin\theta\sigma_R)^2 = 2(\cos\theta\sigma_r)^2 + 2(r\sin\theta\sigma_\theta)^2 \tag{68}$$

where the first term of the left-hand side of Equation 68 is the variance of depth differences (on each beam angle) calculated from the coincident depth measurements obtained from each swath. All other terms are doubled because there are two independent measurements of each.

Estimation of the two parameters (standard deviation of roll and beam angle) may be accomplished using an iterative least squares approach, with a large (statistically significant) number of measurements of depth difference for each beam angle. The parameters may have to be determined for each mode that the MBES uses, as the pulse length may increase with depth, thus decreasing the range measurement accuracy. The MBES system must be in perfect calibration before attempting such a procedure.

Beam angles from either side of nadir can be binned in order to increase the number of degrees of freedom, providing the seafloor is flat. Where significant slope exists, beam angles on either side of nadir should not be binned. Separate tests over different seafloor slopes can be conducted to determine the measurement dependence on slope.

5. POSITION ERROR EQUATIONS

In the following sections, the errors in the right hand side of Equations 21, 22 and their component equations, will be examined. The variance of position can be calculated from the sum of the variances in each coordinate. The radial variance of position for any offset coordinates is given by:

$$\sigma_p^2 = \sigma_{\Delta y}^2 + \sigma_{\Delta x}^2 \tag{69}$$

Combining the two coordinate error contributions in this manner, however, destroys the directional component of the error. Recovering the direction of the semi-major axis of the error ellipse without the covariance of the two coordinates is impossible, because of the simplistic approach taken. This approach gives a radial value for position error commonly used in hydrography, known as distance root-mean-square or *drms*, given by:

$$drms = \sqrt{\sigma_y^2 + \sigma_X^2} \tag{70}$$

A radial position error, *drms* is not a rigorous measure of position error. Because covariances have been neglected, the confidence level of this measure of error dispersion is typically between 63% and 68% depending on the eccentricity of the bivariate normal distribution.

Since the error estimates for antenna position (measured latitude and longitude in Equations 21 and 22) are typically output by the receiver as values in metres in X and Y, these estimates are used directly in the position error budget for each sounding. Presuming that the ellipsoidal radii (M and N) are without error, the errors in each of the coordinate offsets (the terms in the brackets in Equations 21 and 22), due to the error sources which affect them, can be calculated.

Applying propagation of errors to Equations 21 and 22, and combining terms using Equations 69 and 70, gives the following for radial position error (in metres):

$$\begin{aligned} \sigma_p^{(m)} &= \sqrt{\sigma_\varphi^2 + \sigma_\lambda^2} \\ &\approx \sqrt{\sigma_Y^2 + \sigma_X^2 + \sigma_{\Delta y_s}^2 + \sigma_{\Delta x_s}^2 + \sigma_{\Delta y_a}^2 + \sigma_{\Delta x_a}^2 + \sigma_{\Delta y_t}^2 + \sigma_{\Delta x_t}^2} \\ &\approx \sqrt{drms^2 + \sigma_{P_s}^2 + \sigma_{P_a}^2 + \sigma_{P_t}^2} \end{aligned} \tag{71}$$

The covariance term may be known from the output of the positioning algorithm, but for this discussion all covariances are assumed to be zero. All errors are assumed to be normally distributed and uncorrelated (statistical independence).

The radial position variances for the relative offset coordinates, the last three terms in the last line of Equation 71, are discussed in the following sections.

Error in relative sounding position

Applying propagation of errors to Equations 26 and 27, and combining them using Equation 69, gives the following total position variance for the relative coordinates of the sounding from the transducer:

$$\begin{aligned}\sigma^2_{P_s} &= \sigma^2_{\Delta y_s} + \sigma^2_{\Delta x_s} \\ &= \left(\frac{\partial \Delta y_s}{\partial r}\right)^2 \sigma^2_r + \left(\frac{\partial \Delta y_s}{\partial \alpha}\right)^2 \sigma^2_\alpha + \left(\frac{\partial \Delta y_s}{\partial \theta}\right)^2 \sigma^2_\theta + \left(\frac{\partial \Delta y_s}{\partial P}\right)^2 \sigma^2_P \\ &+ \left(\frac{\partial \Delta x_s}{\partial r}\right)^2 \sigma^2_r + \left(\frac{\partial \Delta x_s}{\partial \alpha}\right)^2 \sigma^2_\alpha + \left(\frac{\partial \Delta x_s}{\partial \theta}\right)^2 \sigma^2_\theta + \left(\frac{\partial \Delta x_s}{\partial P}\right)^2 \sigma^2_P\end{aligned} \tag{72}$$

This error can be broken down into its individual contributions. The first component is that due to range errors. The variance of range has components of measurement error and error due to sound speed uncertainty as given by Equation 38. The equation which maps these errors into a radial position error, after some simplifications, is as follows:

$$\sigma^2_{P_{s1}} = (1 - (\cos\theta \cos P)^2)\sigma^2_r \tag{73}$$

Since this is a radial position variance, no information on direction is contained in the result. In fact, all the heading terms have conveniently cancelled.

The radial position variance as a result of heading errors is given by:

$$\sigma^2_{P_{s2}} = r^2 \, (1 - (\cos\theta \cos P)^2) \, \sigma^2_\alpha \tag{74}$$

In terms of relative positioning error, the error in the heading angle, as measured by the gyrocompass, must be quadratically added to the error in the heading misalignment angle of the transducer as determined from a patch test.

The same equation that maps roll error into relative radial position variance of the soundings with respect to the transducer, will also map the beam measurement error, roll alignment error and beam errors due to sound speed variability into a position variance. The equation is as follows:

$$\sigma^2_{P_{s3}} = r^2 \left(1 - (\sin\theta \cos P)^2\right) \sigma^2_\theta \tag{75}$$

The one remaining component is pitch, which can be mapped into a radial position variance by:

$$\sigma^2_{P_{s4}} = \left((r \cos \theta \cos P)^2\right) \sigma^2_P \tag{76}$$

The relative position variance for the sounder system is given by the sum of the above components, as:

$$\sigma^2_{P_s} = \sigma^2_{P_{s1}} + \sigma^2_{P_{s2}} + \sigma^2_{P_{s3}} + \sigma^2_{P_{s4}} \tag{77}$$

Figure 9 shows how the various error sources contribute to the radial position error between the transducer and each sounding. This value must be calculated for each sounding across a swath.

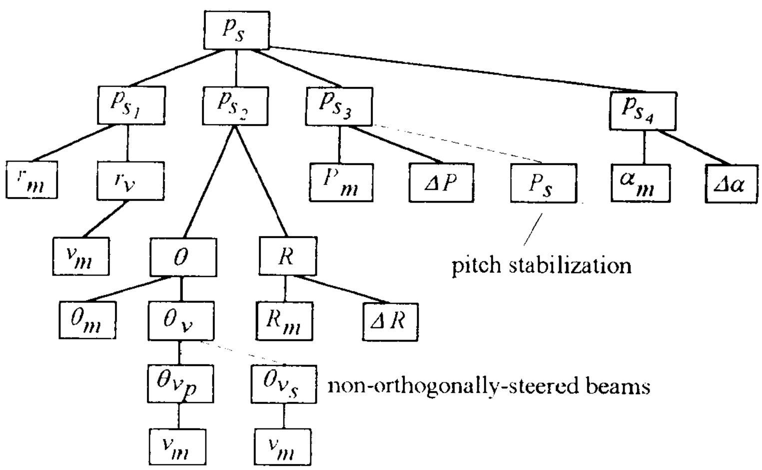

FIG. 9.- Flow diagram showing contributions to relative sounding position error.

Error in relative transducer position

Neglecting covariance terms, propagation of errors applied to Equations 28 and 29 gives the following total position variance for the relative coordinates of the transducer from the positioning system antenna:

$$\sigma^2_{P_a} = \sigma^2_{\Delta y_a} + \sigma^2_{\Delta x_a}$$
$$= \left(\frac{\partial \Delta y_a}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial \Delta y_a}{\partial y}\right)^2 \sigma_y^2 + \left(\frac{\partial \Delta y_a}{\partial z}\right)^2 \sigma_z^2 + \left(\frac{\partial \Delta y_a}{\partial a}\right)^2 \sigma_a^2 + \left(\frac{\partial \Delta y_a}{\partial R}\right)^2 \sigma_R^2 + \left(\frac{\partial \Delta y_a}{\partial P}\right)^2 \sigma_P^2$$
$$+ \left(\frac{\partial \Delta x_a}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial \Delta x_a}{\partial y}\right)^2 \sigma_y^2 + \left(\frac{\partial \Delta x_a}{\partial z}\right)^2 \sigma_z^2 + \left(\frac{\partial \Delta x_a}{\partial a}\right)^2 \sigma_a^2 + \left(\frac{\partial \Delta x_a}{\partial R}\right)^2 \sigma_R^2 + \left(\frac{\partial \Delta x_a}{\partial P}\right)^2 \sigma_P^2 \quad (78)$$

The roll, pitch and heading variances are for sensor measurement and contain no transducer misalignment variance component. The total error can once again be broken down into its individual contributions.

The first component is that due to the offset coordinate measurements. The variances of these measurements would have to be determined by the method that was used to make these measurements - e.g. using a cloth tape may give a standard deviation of +/- 1 cm at 68% confidence for each coordinate. The actual variance of the coordinate differences is the quadratic sum of the errors in each sensor coordinate. The equation which maps these errors into a radial position error, after some simplifications, is:

$$\sigma^2_{P_{a1}} = \left((\cos P)^2\right)\sigma_x^2 + \left((\cos R)^2 + (\sin R \sin P)^2\right)\sigma_y^2 + \left((\sin R)^2 + (\cos R \sin P)^2\right)\sigma_z^2 \quad (79)$$

Note that when combining the x and y components to get a radial position variance, the heading component disappears. This is because a sum of squared sine and cosine terms is equal to 1 and the remaining terms easily combine. As a result, all information about the direction of the error is lost - only its magnitude remains.

The heading error component can be determined from the following:

$$\sigma^2_{P_{a2}} = \begin{pmatrix} (x\cos P)^2 + y^2\left((\cos R)^2 + (\sin R \sin P)^2\right) + z^2\left((\sin R)^2 + (\cos R \sin P)^2\right) \\ -xy\cos P \sin P \sin R - xz \cos P \sin P \cos R - yz(\cos P)^2 \sin R \cos R \end{pmatrix} \sigma^2_{\alpha_m} \quad (80)$$

The relative position error due to antenna-transducer displacement is independent of the transducer, so the variance is of the measured heading, and does not include a component for the transducer heading misalignment.

The roll error component (no beam angle component because the offsets are independent of the sounder system) is mapped into a relative position variance, by:

$$\sigma^2_{P_{a3}} = \begin{pmatrix} y^2\left((\sin R)^2 + (\cos R \sin P)^2\right) + z^2\left((\cos R)^2 + (\sin R \sin P)^2\right) + \\ yz \cos R \sin R (\cos P)^2 \end{pmatrix} \sigma^2_{R_m} \quad (81)$$

There is no x-component of the roll error contribution to relative position error, because the first rotation was about the x-axis (recall Equation 4).

The pitch error component is given by:

$$\sigma^2_{P_{\alpha 4}} = \left((x\sin P + y\sin R\cos P + z\cos R\cos P)^2\right)\sigma^2_{P_m} \tag{82}$$

There are components from all three axis for the pitch error contribution to relative position error, as there was for heading error, because subsequent rotations in Equation 4 involved the other axes.

The total radial, relative position variance due to the offsets of the transducer from the positioning system antenna is given by the sum of the above variances as:

$$\sigma^2_{P_\alpha} = \sigma^2_{P_{\alpha 1}} + \sigma^2_{P_{\alpha 2}} + \sigma^2_{P_{\alpha 3}} + \sigma^2_{P_{\alpha 4}} \tag{83}$$

Error in relative position-time displacement

An error in the knowledge of positioning system time offset (latency) will cause an additional position error in the along-track direction. The radial error can be calculated by applying propagation of errors to Equations 31 and 32 and summing the squared terms to give:

$$\begin{aligned} \sigma^2_{P_t} &= \sigma^2_{\Delta_{y_t}} + \sigma^2_{\Delta_{x_t}} \\ &= \left(\frac{\partial \Delta y_t}{\partial SOG}\right)^2 \sigma^2_{SOG} + \left(\frac{\partial \Delta y_t}{\partial \Delta t}\right)^2 \sigma^2_{\Delta t} + \left(\frac{\partial \Delta y_t}{\partial \alpha}\right)^2 \sigma^2_{\alpha} + \left(\frac{\partial \Delta y_t}{\partial P}\right)^2 \sigma^2_{P} \\ &+ \left(\frac{\partial \Delta x_t}{\partial SOG}\right)^2 \sigma^2_{SOG} + \left(\frac{\partial \Delta x_t}{\partial \Delta t}\right)^2 \sigma^2_{\Delta t} + \left(\frac{\partial \Delta x_t}{\partial \alpha}\right)^2 \sigma^2_{\alpha} + \left(\frac{\partial \Delta x_t}{\partial P}\right)^2 \sigma^2_{P} \end{aligned} \tag{84}$$

The error contributions can again be broken down into separate components. The variance in position due to an error in the speed over ground is given by:

$$\sigma^2_{P_{t_1}} = (\Delta t\cos P)^2\sigma^2_{SOG} \tag{85}$$

The variance in position due to an error in the time offset between the two systems is given by:

$$\sigma^2_{P_{t_2}} = (SOG\cos P)^2\sigma^2_{\Delta t} \tag{86}$$

The variance in position due to an error in the heading is given by:

$$\sigma^2_{P_{t_3}} = (\Delta t\, SOG\cos P)^2\sigma^2_{\alpha} \tag{87}$$

Finally, the variance in position due to an error in the pitch is given by:

$$\sigma^2_{P_{t4}} = (\Delta t\, SOG \sin P)^2 \sigma^2_P \tag{88}$$

The total position variance resulting from a time offset error is calculated by the sum of these variances as:

$$\sigma^2_{Pt} = \sigma^2_{P_{t1}} + \sigma^2_{P_{t2}} + \sigma^2_{P_{t3}} + \sigma^2_{P_{t4}} \tag{89}$$

This error will propagate into the radial position error in the along-track direction (Equations 85-88 have both time and speed components, which give errors only along-track).

Total sounding position error

All the radial position error components propagate into a single position error for each sounding. Figure 10 illustrates how these error sources combine.

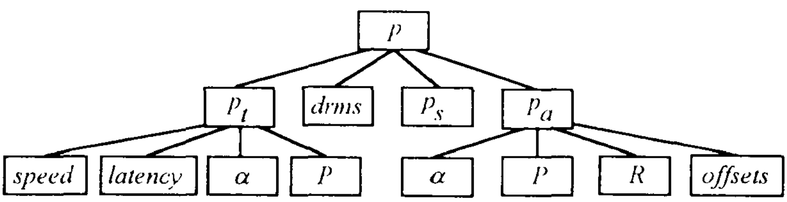

FIG. 10.- Flow diagram showing components of MBES sounding position error.

6. TOTAL ERROR BUDGETS

Depth error budget

Once all of the measurement errors have been transformed into an error in depth, the total error budget for depth can be calculated from the RSS of these depth error contributions. The total error budget for depth is made up of the following components:

1. Sounder system error (range and beam angle measurement errors and beamwidth resolution),
2. Roll error (measurement and misalignment errors),
3. Pitch error (measurement, misalignment and mechanical stabilization errors),
4. Heave error (measured and induced heave errors) and
5. Refraction error (sound speed error effects on range, beam angle and non-orthogonal beam steering).

The RSS of errors 1 to 5 is the error in depth measurement. To this error is quadratically added the following reduction errors:

6. Dynamic Draught error (static draught, squat and loading changes) and
7. Water level error (measurement and spatial prediction).

The RSS of the depth measurement and both depth reduction errors is then given as the error in reduced depth at the 68% confidence level. Multiplying by an expansion factor of 1.96 will bring the error estimate to the 95% confidence level. This confidence level is being proposed to IHO Member States as the depth and position standard for the next edition of S-44.

Position error budget

The total error budget for positioning a sounding on the seafloor is made up of the following components:

1. Positioning system error (e.g. *drms* calculated from standard deviations of latitude and longitude as output from the positioning algorithm),
2. Latency error (errors in the knowledge of positioning system latency),
3. Relative transducer-sounding position error (due to range and beam angle measurement, refraction, roll and pitch measurement, and transducer misalignment errors),
4. Heading error (effect on sounding position from transducer due to gyrocompass measurement error and transducer yaw misalignment),
5. Relative antenna-transducer position error (due to offsets and attitude measurement errors).

The RSS of these errors is then given as the total radial sounding position error, *drms*, or at about the 68% confidence level. The 95% (approximately) confidence value, *2drms*, is obtained by multiplying this number by an expansion factor of 2.

Small angle approximations

In order to simplify the equations given in the previous two sections, some approximations and substitutions can be performed. Small angle approximations assume roll and pitch angles are small enough that the following substitutions can be made, without significant error:

$$\sin(R) = \sin(P) = 0$$

$$\cos(R) = \cos(P) = 1$$

Further simplification can be performed by substituting cross-track distance and depth below transducer from Equations 1 and 2, repeated here for convenience:

$$y = r \sin(\theta)$$

$$d = r \cos(\theta)$$

If the seafloor is not flat, the cross-track distance and depth will have to be calculated for each beam. Finally, by assuming that the positioning system and MBES system are in perfect synchronization, some of the terms in Equation 89 will drop out. The equations below summarize those in Sections 4 and 5, but with small angle approximations, cross-track and depth substitutions, the assumption of zero latency and some simplification.

Total error budgets for MBES systems

The following equations can be used, under most circumstances, to evaluate the total depth and position error budgets of MBES systems. Where larger roll and pitch angles are present, or an unacceptable positioning system latency exists, the full equations of Sections 4 and 5 should be used.

1) Sounder measurement variance (range and beam angle):

$$\sigma_{d1}^2 = \cos\theta^2\, \sigma_r^2 + y^2\, \sigma\theta^2$$

2) Depth variance due to beam opening angle:

$$\sigma_{d2}^2 \approx \left\{ d \left[1 - \cos\left(\frac{\psi}{2}\right)\right]\right\}^2$$

3) Roll variance:

$$\sigma_{d3}^2 = y^2\, \sigma_R^2$$

4) Pitch variance:

$$\sigma_{d4}^2 = 0$$

5) Total heave variance:

$$\sigma_{d5}^2 = \max\left(a^2, (b \times heave)^2\right) + x^2\, \sigma_P^2 + y^2\, \sigma_R^2$$

6) Refraction variance:

$$\sigma_{d6}^2 = \left\{ \left(\frac{d}{v}\right)^2 + y^2 \left[\left(\frac{\tan\theta}{2v}\right)^2 + \left(\frac{\tan(\theta - \delta)}{v}\right)^2 \right] \right\} \sigma_v^2$$

7) Total depth measurement error:

$$\sigma_d = \sqrt{\sigma_{d1}^2 + \sigma_{d2}^2 + \sigma_{d3}^2 + \sigma_{d5}^2 + \sigma_{d6}^2}$$

8) Dynamic draught variance:

$$\sigma_{dyn\_draught}^2 = \sigma_{draught}^2 + \sigma_{squat}^2 + \sigma_{load}^2$$

9) Total reduced depth error:

$$\sigma_D = \sqrt{\sigma_d^2 + \sigma_{dyn\_draught}^2 + \sigma_{WL}^2}$$

10) Total radial position error:

$$\sigma_P = \sqrt{\begin{array}{l} drms^2 \\ +\sin\theta^2\sigma_r^2 + d^2\sigma_\theta^2 + \left\{\left(\dfrac{y}{v}\right)^2 + d^2\left[\left(\dfrac{\tan\theta}{2v}\right)^2 + \left(\dfrac{\tan(\theta-\delta)}{v}\right)^2\right]\right\}\sigma_v^2 + d^2(\sigma_R^2+\sigma_P^2) + y^2\sigma_\alpha^2 \\ +\sigma_x^2 + \sigma_y^2 + (x^2+y^2)\sigma_\alpha^2 + z^2(\sigma_R^2+\sigma_P^2) \\ +\ SOG^2\,\sigma_{\Delta t}^2 \end{array}}$$

The first line of the above equation contains the radial positioning system error. The second line contains all sounder system positioning errors, including refraction and orientation errors. This line should be evaluated for each beam. The third line (relative antenna to transducer position errors) only needs to be calculated once for each MBES vessel. This value can then be quadratically added to positioning error, sounder system error and latency-induced error (fourth line), all calculated in real-time. The x, y and z elements in the third line are the coordinate offsets between the transducer and positioning system antenna (y is not the athwartships sounding coordinate in this line only).

7. CONCLUSIONS AND RECOMMENDATIONS

It was shown that depth and position error equations can be derived for all of the measurement error sources which affect MBES system accuracy. In some cases, the error sources acted in linear combination and were quadratically added. In other cases, a depth or position error component had to be calculated for each error source. Using small angle approximations, the depth and position error component equations were simplified. Total error budget equations were presented for both depth and positions of soundings collected using MBES systems.

At least one MBES manufacturer has developed a range and beam angle measurement error model. Models for depth measurement errors need to be developed, or at least publicized, for other MBES types. Further work on modeling the errors due to uncertain sound speed profiles is needed. MBES manufacturers should be encouraged to implement algorithms which calculate position and depth error estimates in real-time for each sounding. These estimates could be output in the data telegrams and used for quality assurance in real-time, or for more effective data integration in post-mission.

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